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Dual Geometric-Gauge Field Aspects of Gravity

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We propose that the geometric and standard gauge field aspects of gravity are equally essential for a complete description of gravity and can be reconciled. We show that this dualism of gravity resolves the dimensional Newtonian constant problem in both quantum gravity and unification schemes involving gravity (i.e., the Newtonian constant is no longer the coupling constant in the gauge aspect of gravity) and reveals the profound similarity between gravity and other fields.

1. INTRODUCTION

The geometric theory of gravity, general relativity (GR), accounts very well for all known astronomical gravitational phenomena. However, there are two long-standing problems in gravitation physics. First, there is no consistent quantum theory of gravity (Alvarez, 1989). An essential difficulty in the quantization of geometric theories of gravity is caused by the dimensional Newtonian constant G, and leads to nonrenormalizable quantum field theories (Weinberg, 1972; 't Hooft and Veltman, 1974). Second, many disparate approaches to unify gravity with other forces have been proposed without success, because of the geometric nature of GR and its failure to conform to the pattern of the other gauge theories. Fundamental differences between a Yang–Mills theory and GR are:

(A) The structure of the Einstein equation and the corresponding Lagrangian differ from that of a Yang–Mills theory. The interpretations of corresponding quantities, such as $h^{\mu\nu}$ (h^{00} is proportional to the Newtonian potential ϕ_g) and A^μ (A^0 is proportional to the Coulomb potential ϕ_C), are different, although ϕ_g and ϕ_C have the same physical meaning.

(B) Geometric theories of gravity are associated with spacetime groups, while Yang–Mills theories are associated with internal groups.

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(C) The coupling constant G has a dimension, while the coupling constants in other interactions are dimensionless. Facing these difficulties, it has been suggested that (apart from technical developments) there might have to be one or more major conceptual revolutions before the final goal is achieved (Isham, 1975; Duff, 1975; Mills, 1989).

In our view, a fundamental question behind these problems is: What is gravity really? Is gravity a manifestation of the curvature of spacetime, or a standard gauge field like other interactions? How are we to reconcile these two descriptions? Different physicists have different opinions on these questions. This contradictory picture of describing gravity reminds us of the great controversy that arose after de Broglie proposed his matter wave concept: are electrons particles or waves? In this paper we will explore a new avenue to bridge the geometric and gauge field aspects of gravity, resolve the dimensional Newtonian constant problem, and eliminate the fundamental differences between GR and Yang–Mills theories.

2. MAXWELL–YANG–MILLS-TYPE EINSTEIN EQUATION

First, we show that, in spite of its geometric interpretation, the Einstein equation greatly resembles the Maxwell and Yang–Mills equations. This similarity will allow us to follow the example of quantum electrodynamics (QED) step by step, and lead us to a new concept. The Einstein equation,

$$R^{\mu\nu}-\tfrac{1}{2}g^{\mu\nu}R=(8\pi G/c^4)T^{\mu\nu} \tag{1}$$

also has a well-known different form (Weinberg, 1972; Gupta, 1957; Deser, 1970; Wald, 1986):

$$\begin{gathered}\bar{h}^{\mu\nu,\lambda}{}_{\lambda}-\bar{h}^{\mu\lambda,\nu}{}_{\lambda}+\eta^{\mu\nu}\bar{h}^{\lambda\beta,}{}_{\beta\lambda}-\eta^{\mu\lambda}\bar{h}^{\nu\alpha,}{}_{\alpha\lambda}=-(16\pi G/c^4)(T^{\mu\nu}+t^{\mu\nu})\\ g^{\mu\nu}=\eta^{\mu\nu}+\bar{h}^{\mu\nu}-\tfrac{1}{2}\eta^{\mu\nu}\bar{h}^{\alpha}{}_{\alpha}, \qquad (\mu,\nu=1,2,3,4)\end{gathered} \tag{2}$$

where $t^{\mu\nu}$ is the energy-momentum tensor of gravity.

Defining the tensor gravitational field "strength"

$$G^{\mu\nu\lambda}\equiv(1/4)(\bar{h}^{\mu\nu,\lambda}-\bar{h}^{\mu\lambda,\nu}+\eta^{\mu\nu}\bar{h}^{\lambda\alpha,}{}_{\alpha}-\eta^{\mu\lambda}\bar{h}^{\nu\alpha,}{}_{\alpha}) \tag{3}$$

we find that equation (2) yields the Maxwell–Yang–Mills-type equation (Peng, 1983, 1986)

$$\frac{\partial G^{\mu\nu\lambda}}{\partial x^{\lambda}}=-\frac{4\pi G}{c^4}(T^{\mu\nu}+t^{\mu\nu}) \tag{4}$$

The Lagrangian for equation (4) in quadratic terms of $G^{\mu\nu\lambda}$ is

$$L = -\frac{c^4}{16\pi G} G^{\mu\nu\lambda} G_{\mu\nu\lambda} + \bar{h}^{\mu\nu} T_{\mu\nu} + L^{(2)} \tag{5}$$

The $t^{\mu\nu}$ term is given by $L^{(2)}$ (Alvarez, 1989; Gupta, 1957; Deser, 1970; Wald, 1986). To discuss $t^{\mu\nu}$ in detail is beyond the scope of this paper.

Under the weak-field and low-velocity limit, equation (4) reduces to the Newtonian gravity and vector gravitomagnetic equation (Braginski *et al.*, 1977)

$$\nabla \times \mathbf{B}_g^0 \simeq -4\pi G/c^4 \mathbf{T}^0, \qquad \mathbf{B}_g^0 \equiv \tfrac{1}{4} \nabla \times \mathbf{h}^0 \tag{6}$$

Now $\mathbf{h}^0 = (h^{01}, h^{02}, h^{03})$ has a dual interpretation, either as a vector gravitomagnetic potential in equations (4)–(6), or as a portion of the metric tensor in equations (1), (2). The detection of vector gravitomagnetic effects is one of the frontiers in gravitation physics (Wheeler, 1988). It is natural to anticipate that the similarity between the Einstein equation (4) and a Yang–Mills equation is profound, and reflects the intrinsic nature of gravity, i.e., the gauge field aspect of gravity.

3. THE G DILEMMA

We reexpress the difficulties related to G in quantum gravity and unification to help us to see the situation clearly. The Einstein geometric and Yang–Mills-type equations (1) and (4) are in the meter-kilogram-second (MKS) system of units, in which the $h^{\mu\nu}$ are dimensionless as required by the geometric picture of gravity. Let us recall the Maxwell equation in the MKS system (Appendix). Corresponding to G, one has K in the Maxwell theory. Both G and K play similar roles. In order to quantize electrodynamics, the Maxwell equation is written in the Heaviside–Lorentz (HL) system of units, in which the charge q is converted to $Q_e = q(4\pi K)^{1/2}$ and $F^{\mu\nu}/(4\pi K)^{1/2} \to F^{\mu\nu}$. Then K disappears, and the coupling constant is Q_e. When we take $\hbar = c = 1$, Q_e is dimensionless.

If one simply defines $Q_g \equiv m_g(4\pi G)^{1/2}$ as the gravitational mass charge (Hsu, 1979) and coupling constant, then G would disappear, but the geometric interpretation of the Einstein equation would be destroyed, because this replacement would give the metric tensor a dimension. The scalar curvature R is not scale invariant.

Table I summarizes the conflicting situations related to G.

According to the correspondence principle, a satisfactory quantum theory of gravity must reproduce GR at the macroscopic level. Without G,

Table I. The G Dilemma

		With G	Without G
Classical level	Geometric interpretation	Valid	Destroyed
	Correspondence principle	Valid	Violated
Quantum level	Perturbative quantization	Impossible	Possible
	Unify with other forces	Difficult	Possible

the geometric interpretation would be violated. With G as a coupling constant, gravity cannot be quantized perturbatively. Even if finally gravity were to be quantized nonperturbatively (Deser, 1980; Ashtekhar, 1986), there may still be problems in unification, since G (analogous to the Fermi dimensional coupling in the Fermi weak theory) has the potential of causing difficulty in unification (Slansky, 1984). This is indeed a profound dilemma: G is needed at the classical level, but not at the quantum level. We call this "the G dilemma," which indicates that the geometric aspect of gravity alone proves inadequate for the full description of gravity.

4. DUAL GEOMETRIC–GAUGE FIELD ASPECTS OF GRAVITY

We will resolve the G dilemma by rethinking the nature of gravity, i.e., modifying the geometric interpretation of gravity to a certain extent. We introduce a new concept, dual geometric–gauge field aspects of gravity. More specifically, the dualism of gravity states: (1) When one quantizes gravity and unify it with other quantum fields, gravity should be considered as a standard gauge field; (2) when one deals with astronomical phenomena, the geometric description of gravity is the best choice; (3) there are phenomena, such as gravitational waves and the effects of gravitomagnetic fields (or dragging of inertial frames), which can be equally well explained by the geometric and gauge field aspects of gravity; and (4) the two aspects can be reconciled, which ensures the correspondence principle.

The root of the dualism of gravity is actually in the Einstein theory, which associates a physical entity $T^{\mu\nu}$ with geometry R. Thus, there must be a dimensional constant to connect $T^{\mu\nu}$ and R, which is G,

$$\text{geometry } R \overset{G}{\leftrightarrow} \text{energy-momentum } T^{\mu\nu}$$

In contrast, only dimensionless constants are needed to couple physical currents and physical fields in a Yang–Mills theory,

$$\text{gauge fields} \overset{g}{\leftrightarrow} \text{currents } J^{i\mu}$$

Table II. Dual Aspects of Light and Gravity

	Light	Gravity
Newtonian scheme	Newtonian particle	Newtonian force
Classical level	Wave	Geometry
Quantum level	Particle–wave	Gauge field–geometry

With this structure, electroweak and strong forces are quantized and unified. Since no one has been able to find a geometric model for the description of matter, a possible way to cure the G dilemma is to consider gravity as a gauge field at the quantum level. The dualism of gravity is an analogue of the dual wave–particle properties of light (Table II).

5. A RESOLUTION OF THE G DILEMMA AND RECONCILIATION OF THE TWO ASPECTS OF GRAVITY

To test the dual concept, we apply it to resolve the G dilemma. The structure and interpretation of physical laws are independent of the choice of the system of units. The statement (1) of the dualism of gravity requires us to treat the Einstein equation (4) as a physical law and allows us to change the system of units to a convenient one regardless of the unit of $\bar{h}^{\mu\nu}$, i.e., without considering the geometric interpretation. Considering the gauge field aspect of gravity, we convert the gravitational mass charge m_g to $Q_g \equiv m_g(4\pi G)^{1/2}$, unit $[Q_g] = m\sqrt{N}$, and $G^{\mu\nu\lambda}/(4\pi G)^{1/2} \to G^{\mu\nu\lambda}$. Equations (4) and (5) become, respectively,

$$\frac{\partial G^{\mu\nu\lambda}}{\partial x^\lambda} = -\frac{1}{c^2}(T^{\mu\nu} + t^{\mu\nu}) \tag{7}$$

$$L = -\frac{1}{4} G^{\mu\nu\lambda} G_{\mu\nu\lambda} + \frac{1}{c^2} \bar{h}^{\mu\nu} T_{\mu\nu} + L^{(2)} \tag{8}$$

where G no longer exists. The coupling constant which couples energy-momentum and the gravitational gauge field is Q_g,

$$\text{gravitational gauge field} \overset{Q_g}{\leftrightarrow} \text{energy-momentum } T^{\mu\nu}$$

When take $\hbar = c = 1$, Q_g is dimensionless. The Lagrangian in quadratic terms of the field strength $G^{\mu\nu\lambda}$ might cure the divergence problem in quantization (DeWitt, 1965; Yang, 1974; Szczyrba, 1987). Equations (7) and (8) form a basis for quantum gravitodynamics and unification involving gravity.

In this new system of units, called the gravitational Heaviside–Lorentz (GHL) system of units, $\bar{h}^{\mu\nu}$ has the unit of potential, which should not surprise us, because h^{00} is proportional to the Newtonian potential, h^{0i} is the vector gravitomagnetic potential, and h^{ij} describe gravitational waves. This leads us to define $h^{\mu\nu}$ as the gauge potential. In contrast, the tetrad and affine connection [they are not dynamically independent gauge potentials at all (Isham, 1975; Duff, 1975; Mills, 1989)] have been defined as gauge potentials in some gauge theories of gravity associated with spacetime groups.

The extension from Newtonian gravity to the Maxwell–Yang–Mills-type gravitation equation (7) resembles very much the extension from the Coulomb law to the Maxwell theory and to the Yang–Mills theory. The comparison between the Maxwell theory and equations (7) and (8) (Appendix) shows the following:

1. The profound similarity between the gauge field aspect of gravity and the electromagnetic field is revealed in the HL system of units: not only do the field equations and the Lagrangians have similar form, but the corresponding quantities have the same units; and thus the same physical interpretations, such as coupling constants Q_g and Q_e, gauge potentials $h^{\mu\nu}$ and A^{μ}, and field strengths $G^{\mu\nu\lambda}$ and $F^{\mu\nu}$. The fundamental differences (A) and (C) mentioned in the Introduction are eliminated.

2. Equations (7), (4), and (1) are mathematically equivalent, but the geometric interpretation is no longer valid in the gauge field aspect of gravity in the GHL system of units, which is the *only price* we pay for resolving the G dilemma without destroying the geometric aspect of gravity in the MKS system of units. Of course, when we convert back to the MKS system, G will be restored, the gravitational gauge field equation (7) will be converted to the Einstein equations (4) and (1) with geometric interpretation, and all of the results obtained in the GHL system of units can be converted to the results of Einstein geometric equation.

3. The weak equivalence principle (WEP) in the gauge aspect is still valid and states now that the gravitational and inertial masses are equal; the ratio of gravitational mass charge Q_g to the gravitational mass m_g is a constant, $(4\pi G)^{1/2}$; particles with different masses in a gravitational field have the same acceleration. Einstein's elevator still works.

6. CONVERSION BETWEEN SPACETIME AND INTERNAL SYMMETRIES

As mentioned above, the geometric aspect of gravity is associated with a spacetime symmetry, while a Yang–Mills gauge theory is associated with internal symmetry. If the dualism of gravity is intrinsic at all, it must satisfy a criterion that the gravitational gauge field should be associated with an internal symmetry, and that the internal symmetry should be able to be converted to a spacetime symmetry in the geometric aspect of gravity.

At the classical level, the sources of gravity are just the energy and momentum distributions associated through Noether's theorem with the spacetime translation group. It has been claimed that a successful extension of GR to the quantum level must take into account spin; thus, the symmetry group should be the Poincaré group. It is not our purpose here to select a specific group for classical or quantum gravity. Rather, we show, by analyzing an example, that the dualism of gravity is capable of satisfying this criterion as long as the generators of symmetry groups contain the gravitational mass charge Q_g.

Let us consider the time translation group T^0 with generator p^0, which produces the transformation

$$\exp[(i/\hbar)P^0 a_0], \qquad x_0' = x_0 + a_0 \qquad (x_0 = ct) \tag{9}$$

where a_0 is the time displacement. In the MKS system of units,

$$\text{unit } [p_0 a_0/\hbar] = \text{unit } [m_i c^2 t/\hbar] = \text{unit } [m_g c^2 t/\hbar] \quad \text{(WEP)} \tag{10}$$

When we consider the gauge field aspect, we convert to the GHL system of units. The nature of the symmetry of gravity will not change. The unit of the generator is changed [absorbing $(4\pi G)^{1/2}$ into P^0]. To keep the argument of the exponential dimensionless, the displacement a^0 has to be changed to $a^0/(4\pi G)^{1/2}$. In the GHL system of units, we have

$$\text{unit } [m_g c^2 t/\hbar] \equiv \text{unit } [Q_g \theta_g/\hbar] \tag{11}$$

where

$$\theta_g \equiv c^2 t/(4\pi G)^{1/2} \qquad \text{and} \qquad \text{unit } [\theta_g] = \sqrt{N}\, s \tag{12}$$

To see what θ_g is, now let us compare θ_g with the internal space of QED. Since both the gravitational and electric charges Q_g and Q_e have the same unit and are the generators of the internal symmetry groups of gravity and QED, respectively, we expect that the coordinates of both internal spaces will have the same unit. To show this, let us recall the $U(1)$ group of QED, which is a transformation (Yang, 1980), $\exp[(i/\hbar)Q_e\theta_e]$. Then we have

$$\text{unit } [\theta_e] = \sqrt{N}\, s \tag{13}$$

which is the same as that of θ_g, as expected. Therefore the internal spaces of both QED and gauge field gravity may be unified into a higher-dimension internal space. The T^0 is a dual group in the sense that it presents both a time translation in the geometric aspect of gravity and a timelike translation in the internal space in the gauge field aspect of gravity. In other words, the internal translation $T^0_{\rm int}$ converts to the time translation T^0, and vice versa. When we convert to the MKS system of units, the internal space associated with gravity (or QED) will (or will not) be converted into spacetime. A similar phenomenon, of isospin converting into spin, occurs in a quantum system (Jackiw and Rebbi, 1976; Li, 1985). Therefore, from the point of view of gauge theory, the index μ of $G^{\mu\nu\lambda}$ and $h^{\mu\nu}$ is also a group index, i.e., it is a dual index. For consistency, we will use $G^{i\nu\lambda}$ and $h^{i\nu}$ in the GHL system.

We have shown that the fundamental differences mentioned above are actually between the geometric aspect of gravity and a Yang–Mills theory, and that there are no such fundamental differences at all between a Yang–Mills theory and the gauge field aspect of gravity. This supports our assumption that the standard Yang–Mills gauge field aspect is indeed the intrinsic nature of gravity, and suggests a simpler way for unification involving gravity.

7. DISCUSSION

Gravity exhibits both geometric and standard gauge field aspects. One of the two aspects alone proves inadequate for the full description, but the two aspects together fully explain gravity. The way we treat gravity as a gauge field is quite different from that of other gauge theories of gravity. We completely separate the geometric and gauge field aspects of gravity, and then show how one aspect can be converted into another, while other gauge theories combine the gauge principle and Einstein's covariant principle (with the Newtonian constant G) throughout the whole treatment. The GHL system of units is more appropriate than the MKS unit system for describing the gauge field aspect of gravity. We anticipate that gravity can be quantized perturbatively, based on three reasons: (1) gravity is the weakest force; (2) the coupling constant Q_g is dimensionless ($\hbar = c = 1$); and (3) the Lagrangian is in a quadratic term of $G^{i\mu\nu}$. The facts that the G dilemma can be resolved by the dualism of gravity, and that the proposed similarity between the gauge field aspect of gravity and Yang–Mills gauge fields is revealed, indicate that the geometric–gauge field dualism of gravity contains the ingredients for a satisfactory description of gravity, and will prove fruitful for further developments.

APPENDIX 1. COMPARISON BETWEEN EINSTEIN AND MAXWELL EQUATIONS

Maxwell	Einstein
Geometric equations	
MKS units	
	$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=(8\pi G/c^4)T^{\mu\nu}$
	$L=-\frac{c^4}{16\pi G}\sqrt{g}\,R+L_{\text{matter}}$
Gauge field equations	
MKS units	
$\frac{\partial F^{\nu\lambda}}{\partial x^\lambda}=\frac{4\pi K}{c^2}J^\nu$	$\frac{\partial G^{\mu\nu\lambda}}{\partial x^\lambda}=-\frac{4\pi G}{c^4}(T^{\mu\nu}+t^{\mu\nu})$
$F^{\nu\lambda}=A^{\nu,\lambda}-A^{\lambda,\nu}$	$G^{\mu\nu\lambda}=\frac{1}{4}(\bar{h}^{\mu\nu,\lambda}-\bar{h}^{\mu\lambda,\nu})$
$L=-\frac{c^2F^{\nu\lambda}F_{\nu\lambda}}{16\pi K}-J^\mu A_\mu$	$L=-\frac{c^4G^{\mu\nu\lambda}G_{\mu\nu\lambda}}{16\pi G}+\bar{h}^{\mu\nu}T_{\mu\nu}+L^{(2)}$
$F=Kqq/r^2$	$F=Gm_gm_g/r^2$
unit $[q]=\bar{c}$	unit $[m_g]=\text{kg}$
unit $[A^\mu]=\text{m N}/\bar{c}$	unit $[h^{\mu\nu}]=1$
$U(1)$: $\exp[(i/\hbar)q\theta_e]$	T^0: $\exp[(i/\hbar)p^0x_0]$
unit $[\theta_e]=\text{m N s}/\bar{c}$	unit $[x_0]=\text{m}$
	Heaviside–Lorentz units
$\frac{\partial F^{\nu\lambda}}{\partial x^\lambda}=\frac{1}{c}J^\nu$	$\frac{\partial G^{i\nu\lambda}}{\partial x^\lambda}=-\frac{1}{c^2}(T^{i\nu}+t^{i\nu})$
$L=-\frac{1}{4}F^{\nu\lambda}F_{\nu\lambda}-\frac{1}{c}J^\mu A_\mu$	$L=-\frac{1}{4}G^{i\nu\lambda}G_{i\nu\lambda}+\frac{\bar{h}^{i\nu}T_{i\nu}}{c^2}+L^{(2)}$
$F=Q_eQ_e/4\pi r^2$	$F=Q_gQ_g/4\pi r^2$
unit $[Q_e]=\text{m}\sqrt{\text{N}}$	unit $[Q_g]=\text{m}\sqrt{\text{N}}$
unit $[A^\mu]=\sqrt{\text{N}}$	unit $[\bar{h}^{i\nu}]=\sqrt{\text{N}}$
$U(1)$: $\exp[(i/\hbar)Q_e\theta_e]$	T^0: $\exp[(i/\hbar)Q_g\theta_g]$
unit $[\theta_e]=\text{s}\sqrt{\text{N}}$	unit $[\theta_g]=\text{s}\sqrt{\text{N}}$

c̄, Coulomb; m, meter; N, Newton; s, second; kg, kilogram.

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